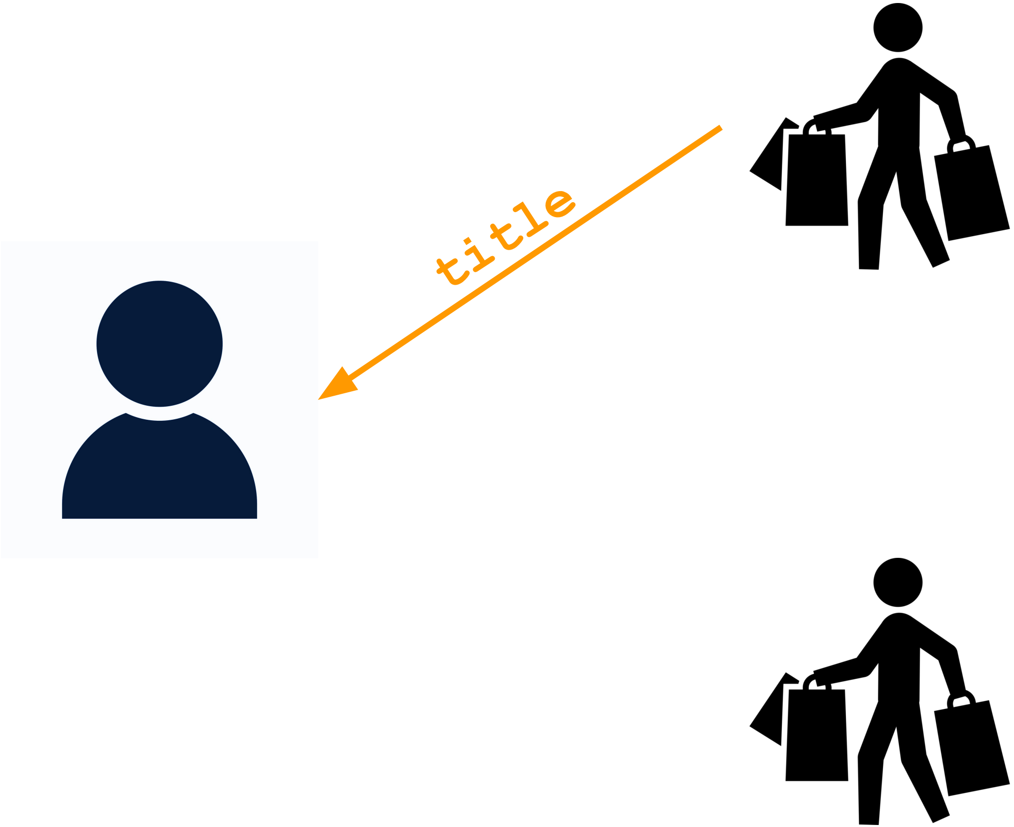
Deciding Subtyping for Asynchronous Multiparty Sessions

Elaine Li Felix Stutz Thomas Wies

NYU NYU

Multiparty session types: two-buyer protocol



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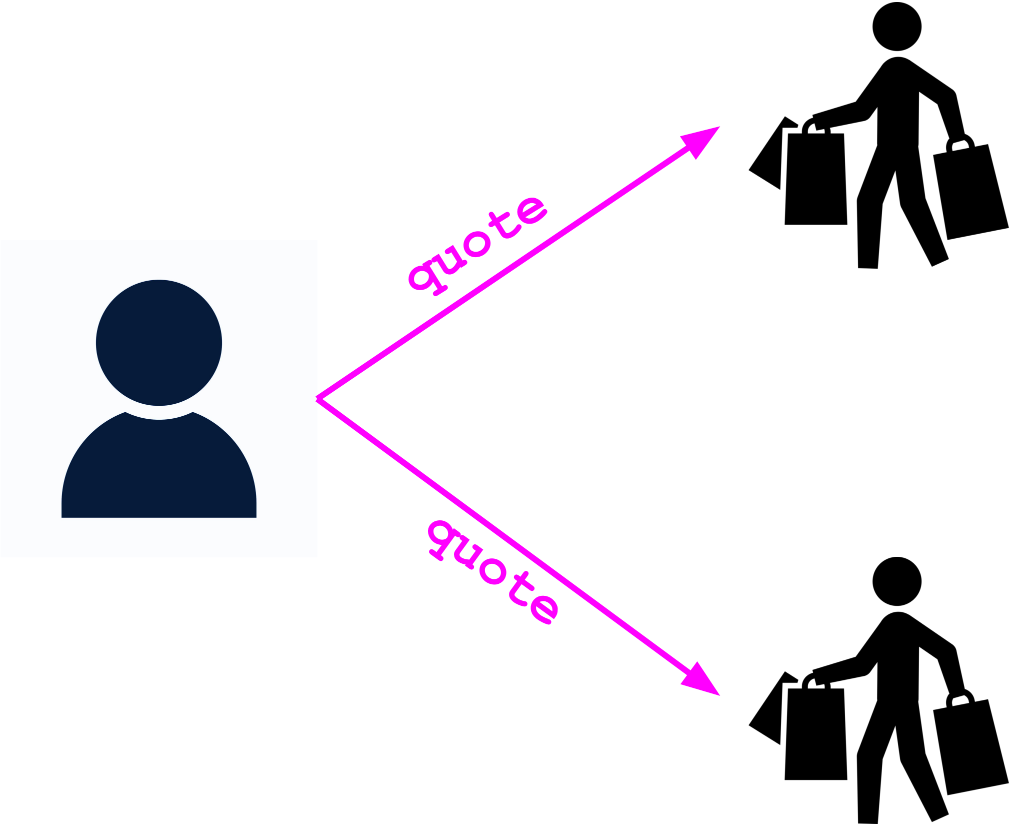

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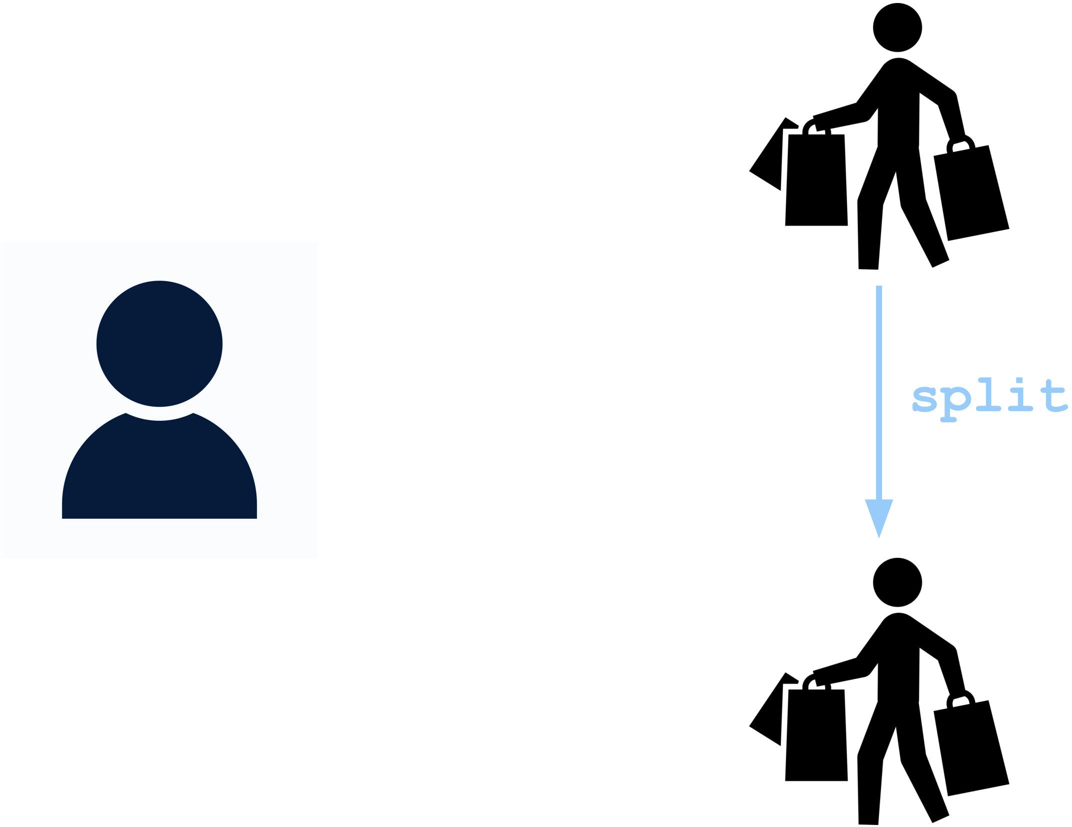

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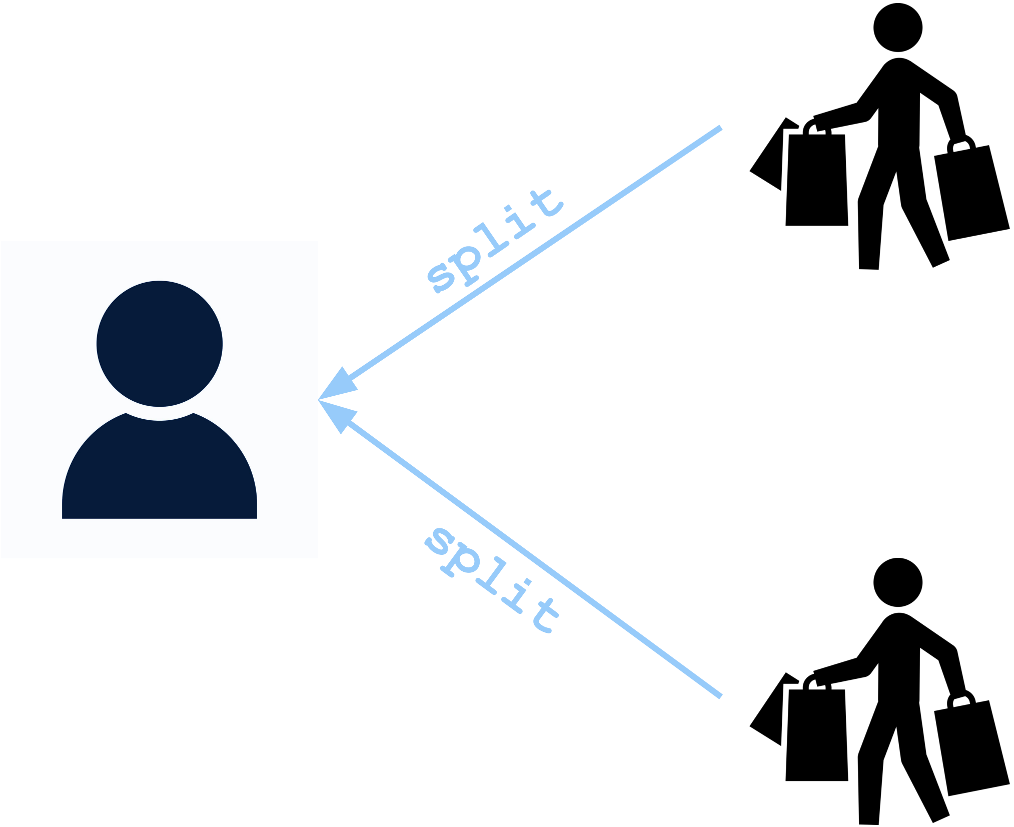

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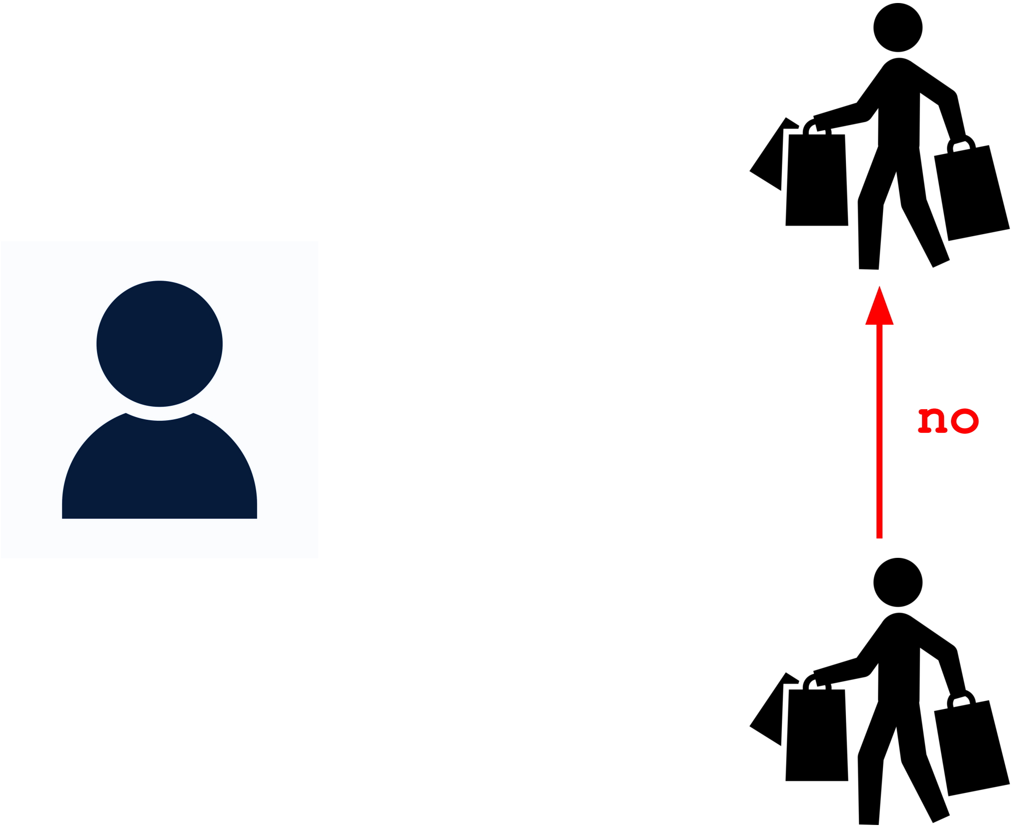

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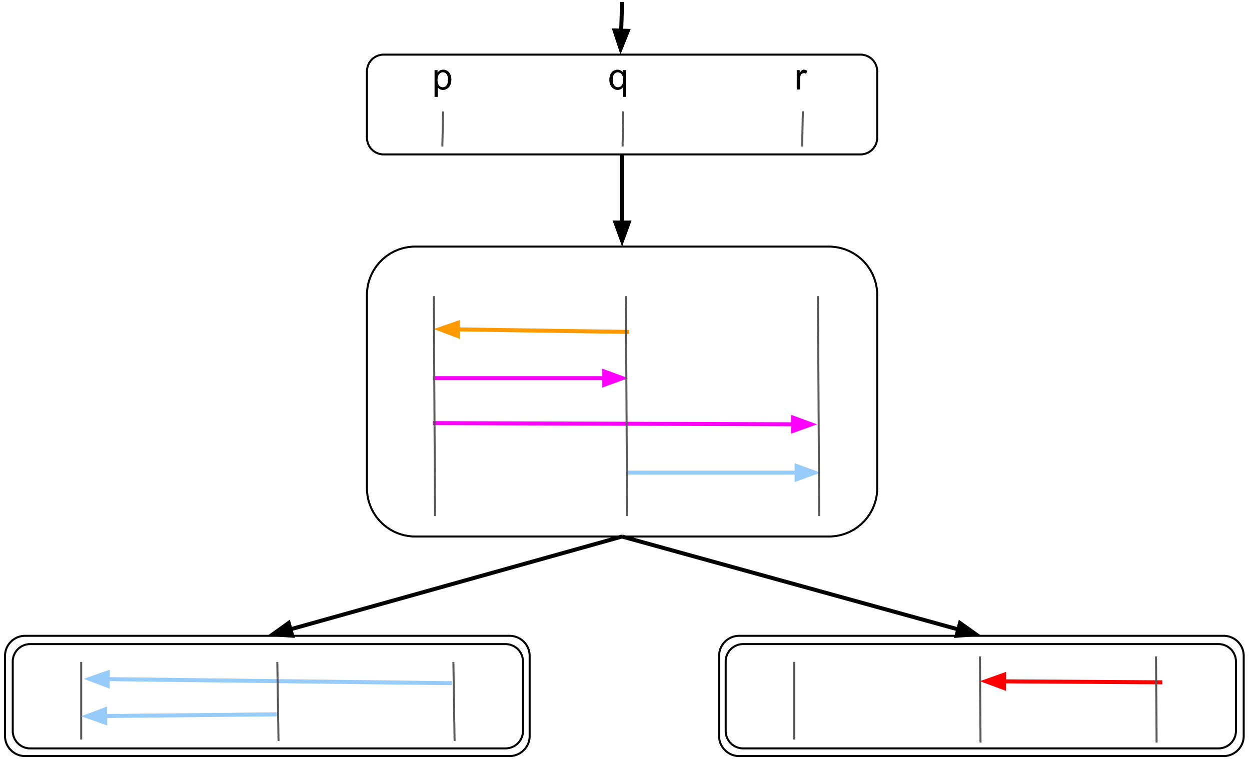
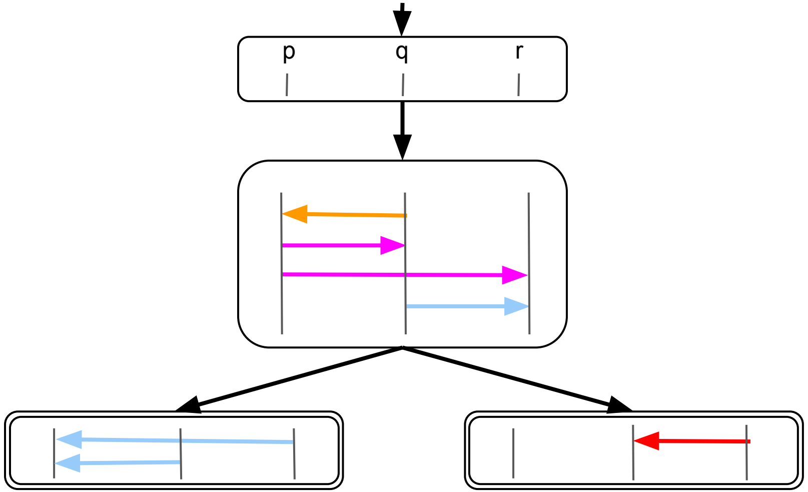


MST semantics



Synchronous $\mathtt{q}{\rightarrow}\mathtt{p}{:}o \cdot \mathtt{p}{\rightarrow}\mathtt{q}{:}m \cdot \mathtt{p}{\rightarrow}\mathtt{r}{:}m \cdot \mathtt{q}{\rightarrow}\mathtt{r}{:}b \ldots$

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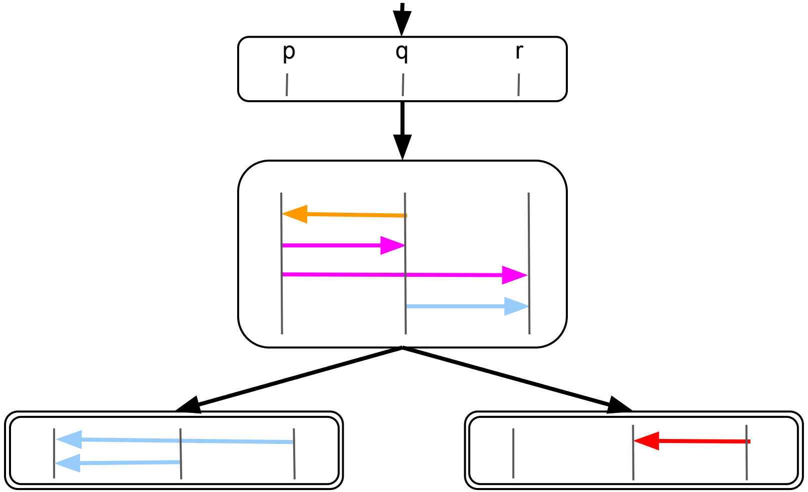
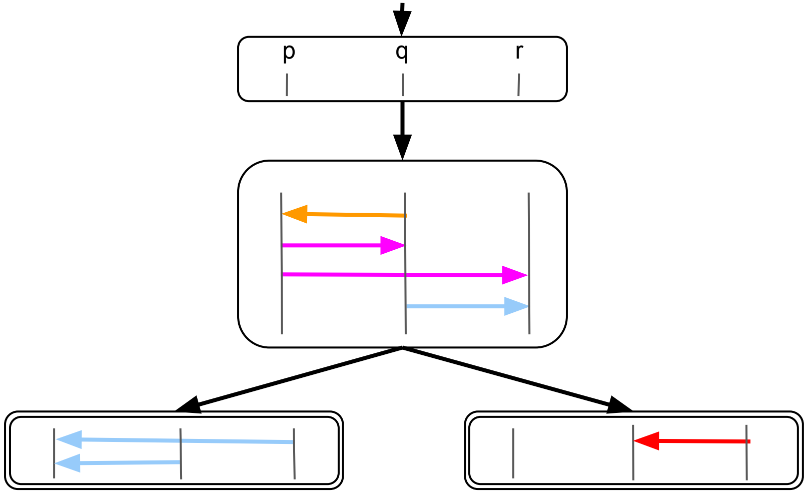


Synchronous $\mathtt{q}\to\mathtt{p}:o \cdot \mathtt{p}\to\mathtt{q}:m \cdot \mathtt{p}\to\mathtt{r}:m \cdot \mathtt{q}\to\mathtt{r}:b \ldots$

Asynchronous $\mathtt{q} \triangleright \mathtt{p}!o \cdot \mathtt{p} \triangleleft \mathtt{q}?o \cdot \mathtt{p} \triangleright \mathtt{q}!m \cdot \mathtt{q} \triangleleft \mathtt{p}?m \cdot \mathtt{p} \triangleright \mathtt{r}!m \cdot \mathtt{r} \triangleleft \mathtt{p}?m \cdot \mathtt{q} \triangleright \mathtt{r}!b \cdot \mathtt{r} \triangleleft \mathtt{q}?b \ldots$

MST semantics



Synchronous $\mathtt{q} \to \mathtt{p} : o \cdot \mathtt{p} \to \mathtt{q} : m \cdot \mathtt{p} \to \mathtt{r} : m \cdot \mathtt{q} \to \mathtt{r} : b \ldots$

Asynchronous $\mathtt{q} \triangleright \mathtt{p}!o \cdot \mathtt{p} \triangleleft \mathtt{q}?o \cdot \mathtt{p} \triangleright \mathtt{q}!m \cdot \mathtt{q} \triangleleft \mathtt{p}?m \cdot \mathtt{p} \triangleright \mathtt{r}!m \cdot \mathtt{r} \triangleleft \mathtt{p}?m \cdot \mathtt{q} \triangleright \mathtt{r}!b \cdot \mathtt{r} \triangleleft \mathtt{q}?b \ldots$

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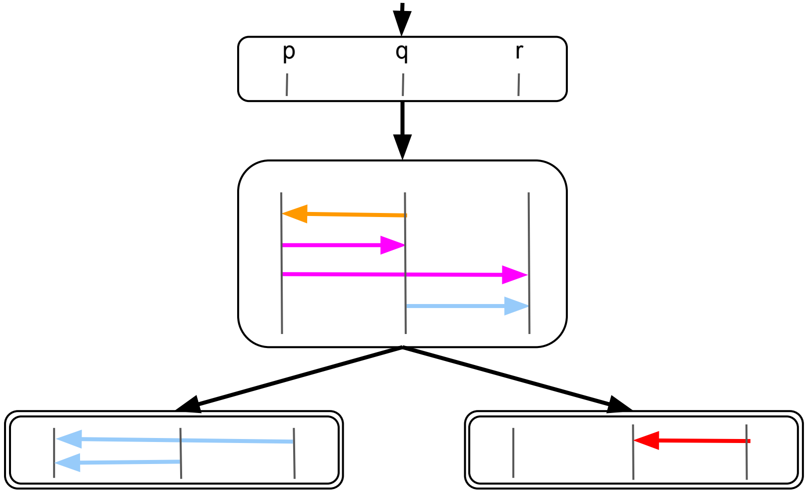

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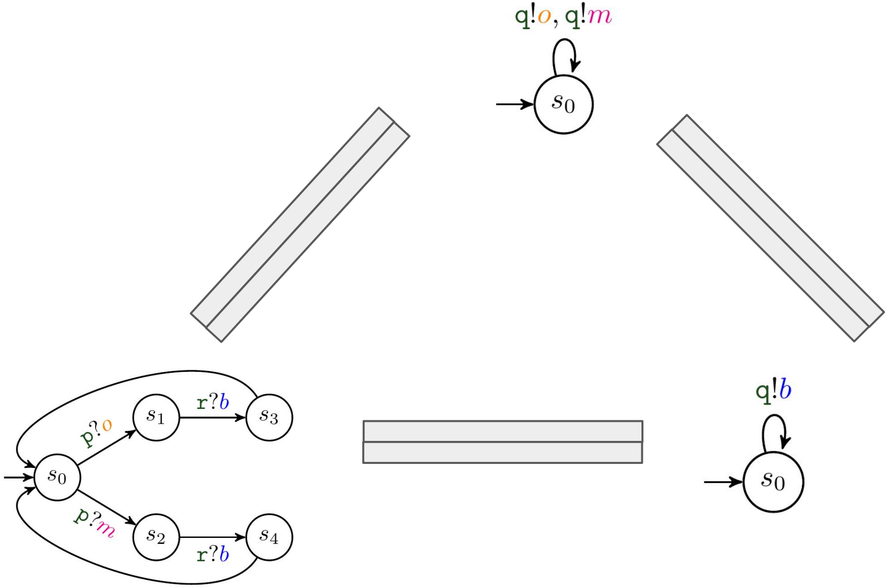
Interleaving $\mathtt{q} \triangleright \mathtt{p}!o \cdot \mathtt{p} \triangleleft \mathtt{q}?o \cdot \mathtt{p} \triangleright \mathtt{q}!m \cdot \mathtt{p} \triangleright \mathtt{r}!m \cdot \mathtt{q} \triangleleft \mathtt{p}?m \cdot \mathtt{r} \triangleleft \mathtt{p}?m \cdot \mathtt{q} \triangleright \mathtt{r}!b \cdot \mathtt{r} \triangleleft \mathtt{q}?b \ldots$

MST implementations

Communicating State Machines (CSM) [Brand and Zafiropulo, JACM'83]

CSM configurations **(s, ξ)** contain **global state** and **channel state**

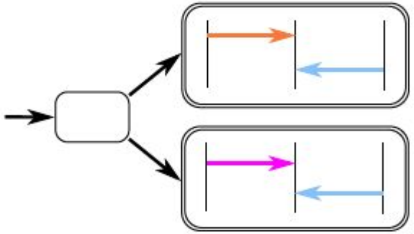


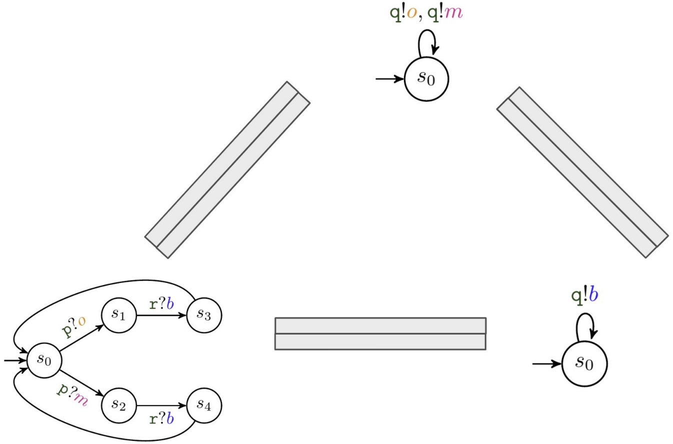
[p := s_0,
q := s_4,
r := s_0]
[(p,q) := ε,
(p,r) := ε,
(q,r) := ε,
(q,p) := ε,
(r,p) := o,
(r,q) := ε]

$\mathtt{p} \triangleright \mathtt{q}!o \cdot \mathtt{q} \triangleleft \mathtt{p}?o \cdot \mathtt{r} \triangleright \mathtt{q}!b \cdot \mathtt{q} \triangleleft \mathtt{r}?b \cdot \mathtt{p} \triangleright \mathtt{q}!m \cdot \mathtt{q} \triangleleft \mathtt{p}?m \cdot \mathtt{r} \triangleright \mathtt{q}!b \cdot \mathtt{q} \triangleleft \mathtt{r}?b \ldots$

MST implementability

Implementability = exists a CSM, protocol fidelity + deadlock freedom





1) CSM language = global type language
2) CSM is deadlock-free

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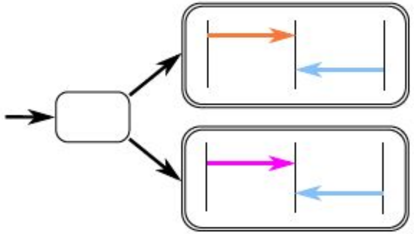
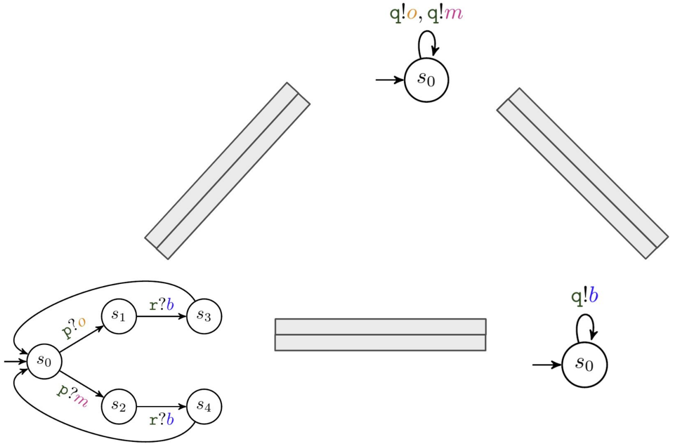


?

first sound and complete projection operator [CAV'23]



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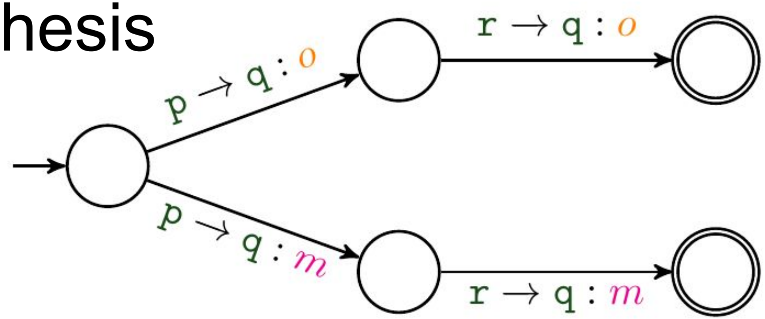
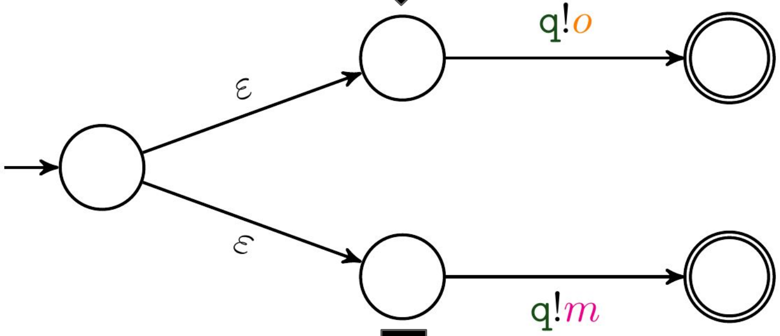
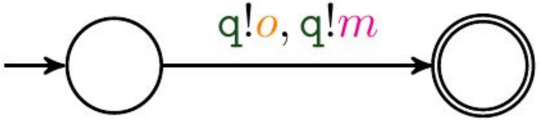
Projection: Synthesis



homomorphism for r

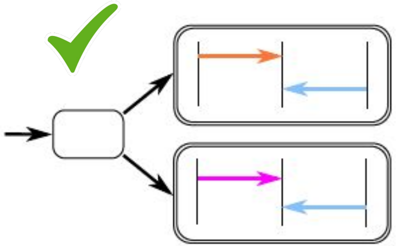


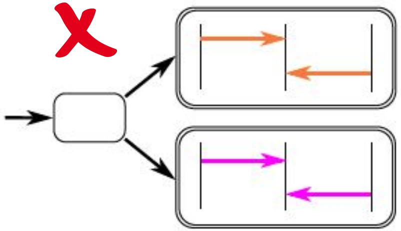
determinization



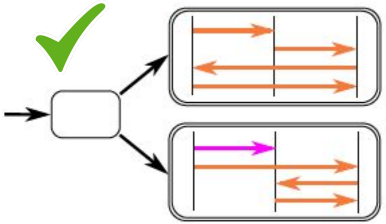
Projection: Checking Implementability

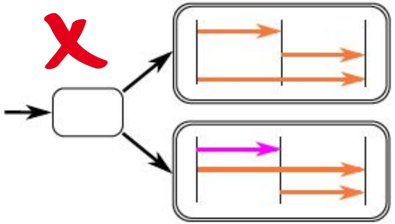
1. Send Validity: "send transitions originate from all global states"





2. Receive Validity: "receive transitions uniquely identify a global state"





MST implementability

Implementability = exists a CSM, protocol fidelity + deadlock freedom

$\mathbf{G(}\{0,1\}^*0\{0,1\}^{n-1}\mathbf{)}$ $\xrightarrow{?}$

first sound and complete projection operator [CAV'23]

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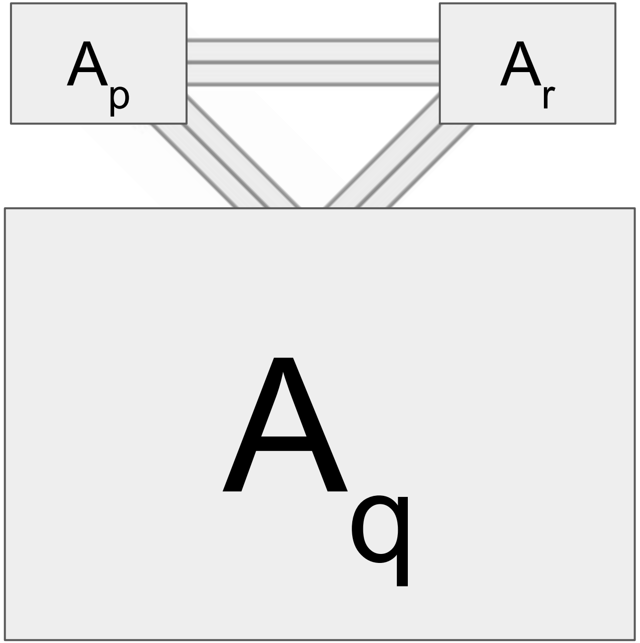

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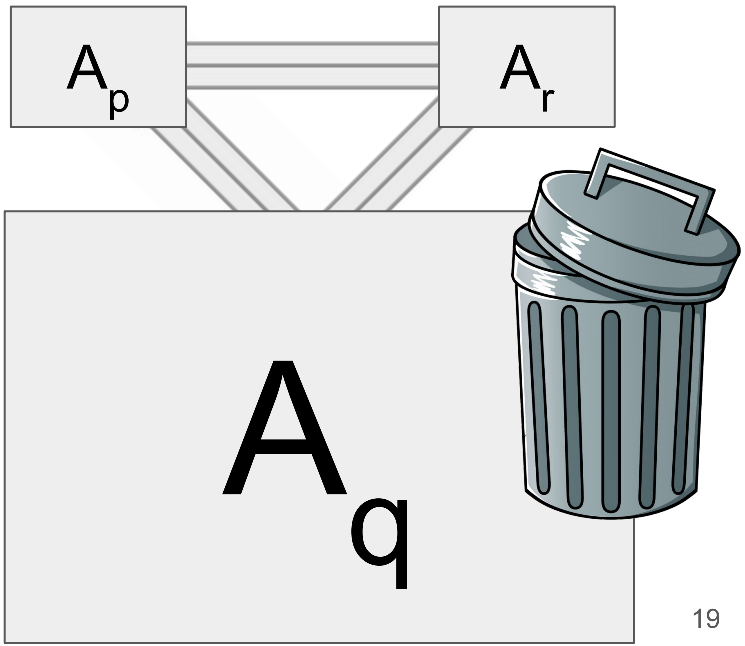
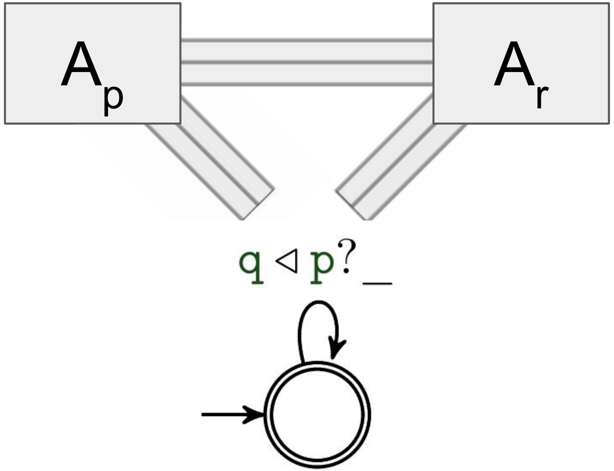


MST protocol verification

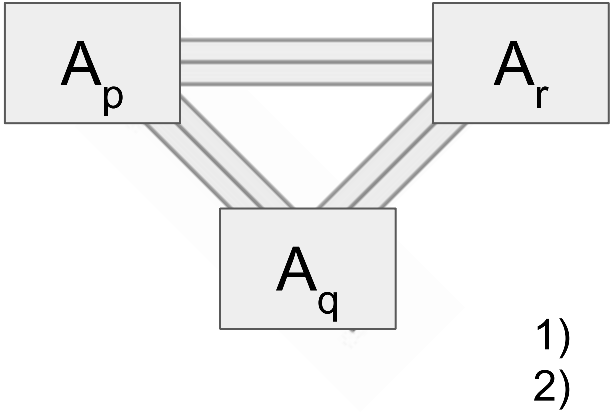
$\mathbf{G(}\{0,1\}{*}0\{0,1\}^{n-1}\mathbf{)}$

implements

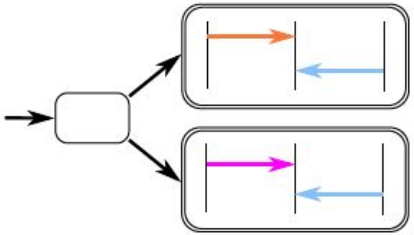


MST protocol verification

Protocol verification = *given* a CSM, protocol fidelity + deadlock freedom



implements?



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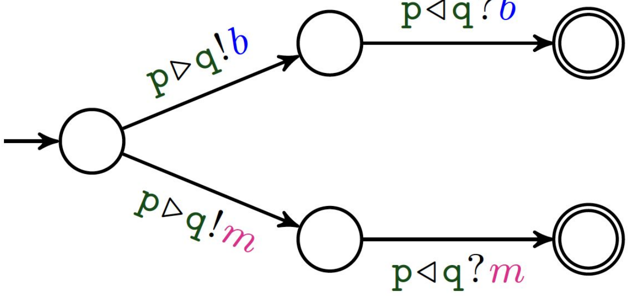

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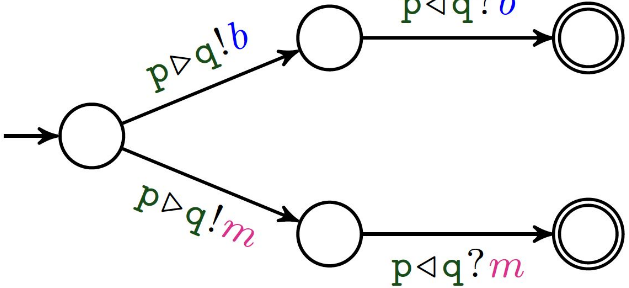
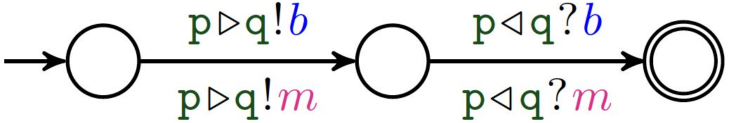


Collapsing states



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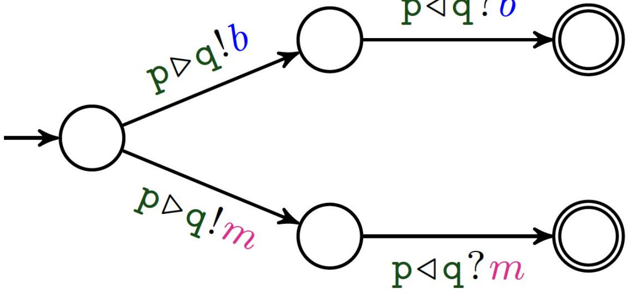
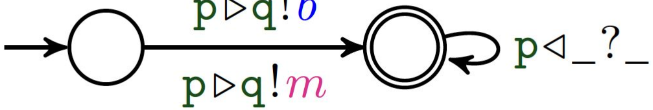
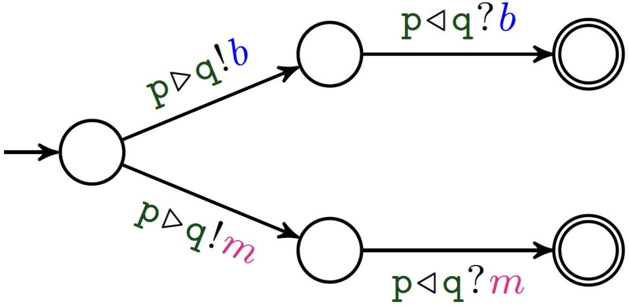


Collapsing states

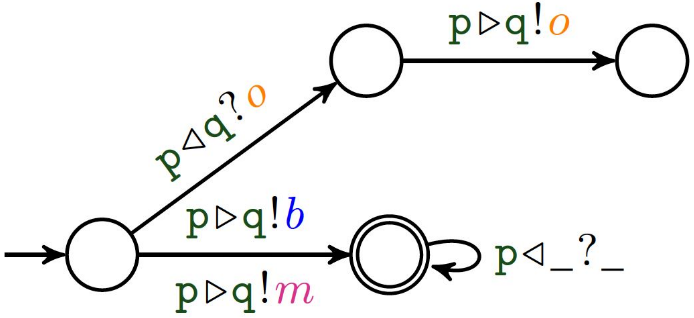


Adding receives

MST protocol verification



Collapsing states



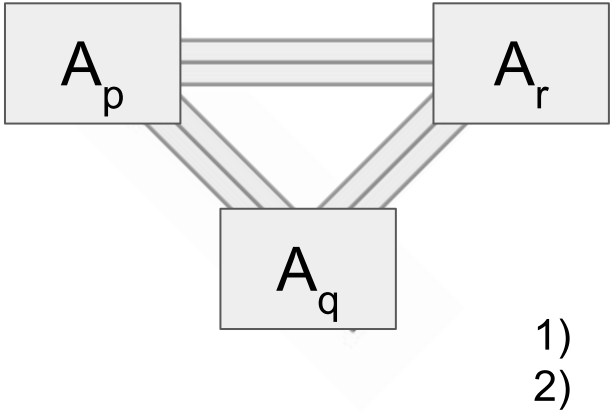
Adding unreachable states

Adding receives

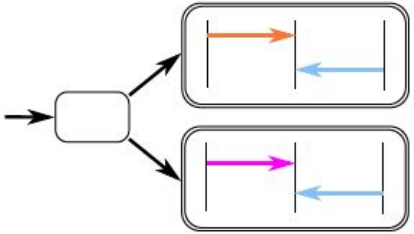
MST protocol verification

decidable in polynomial time, compositional

Protocol verification = *given* a CSM, protocol fidelity + deadlock freedom



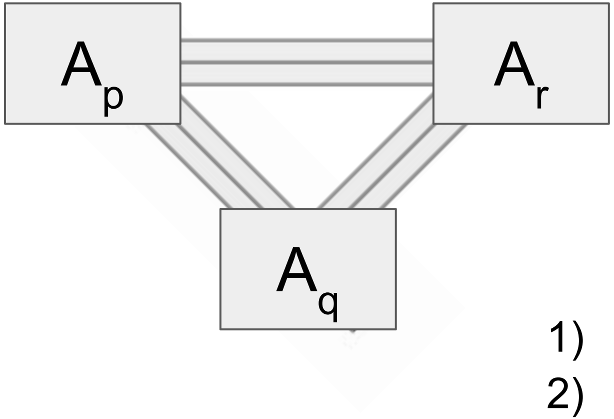
implements?



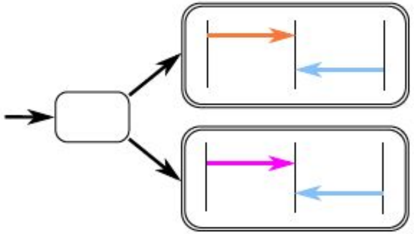
1) CSM language = global type language
2) CSM is deadlock-free

MST monolithic protocol refinement

Monolithic protocol refinement = *given* a CSM, *sub*protocol fidelity + deadlock freedom



protocol refines?



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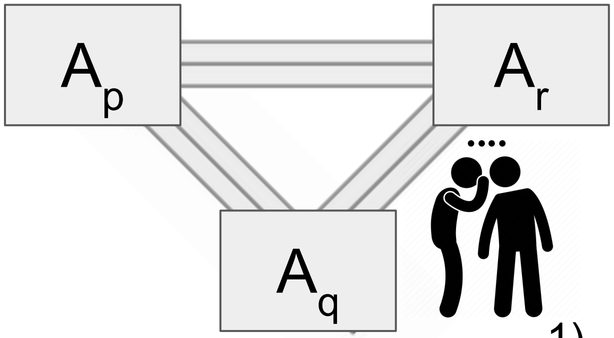

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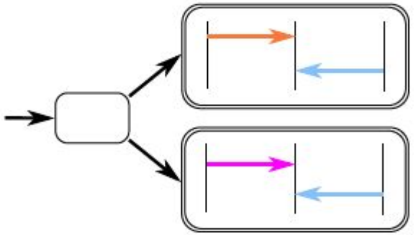

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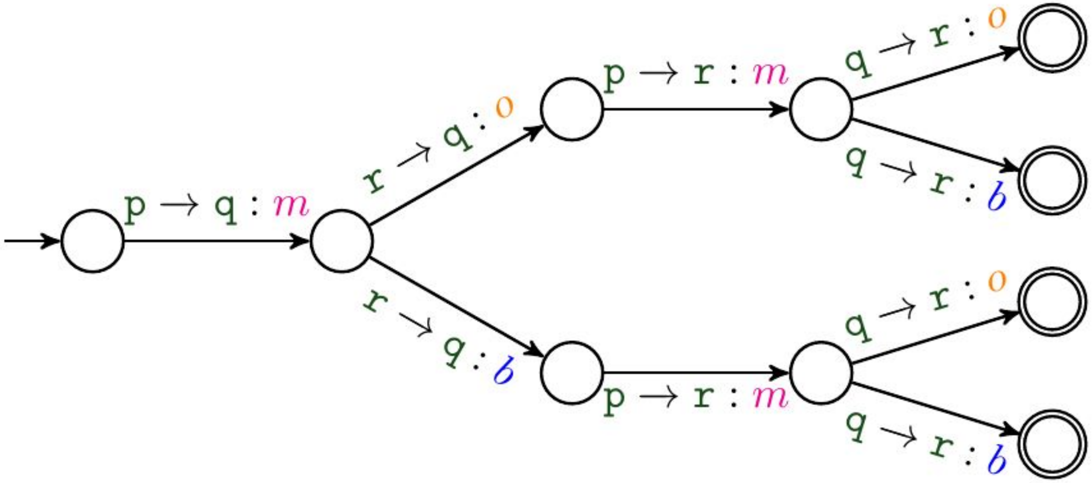
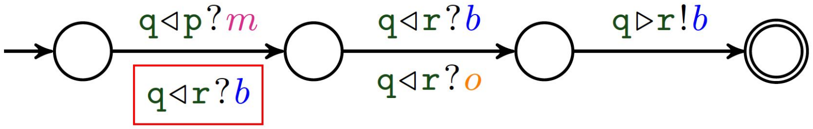
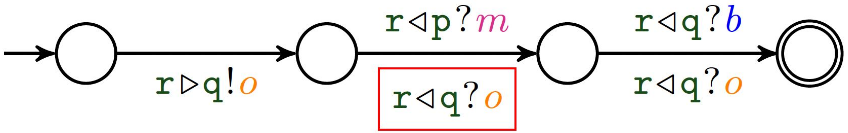


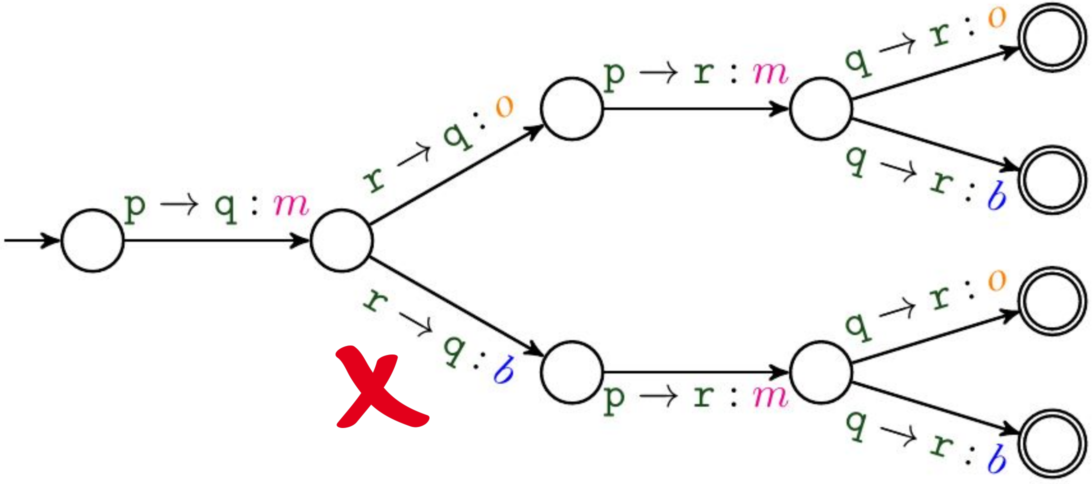
A_q



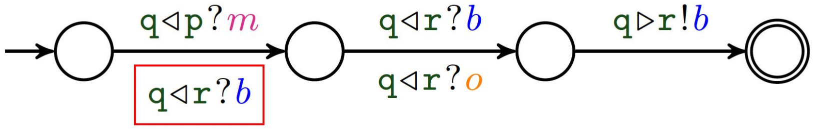
A_r



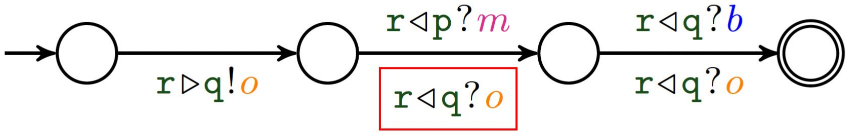
MST monolithic protocol refinement



A$_q$



A$_r$



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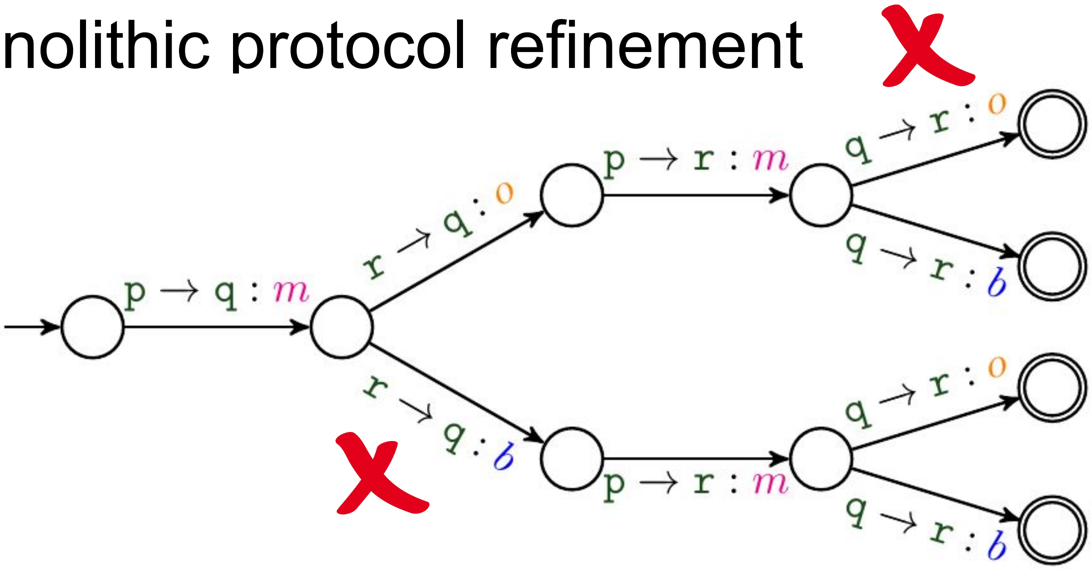
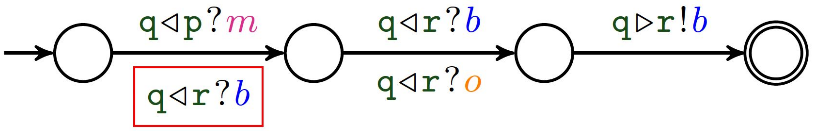
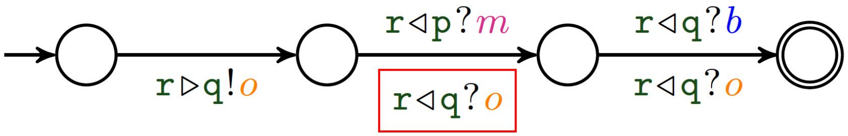


A_q



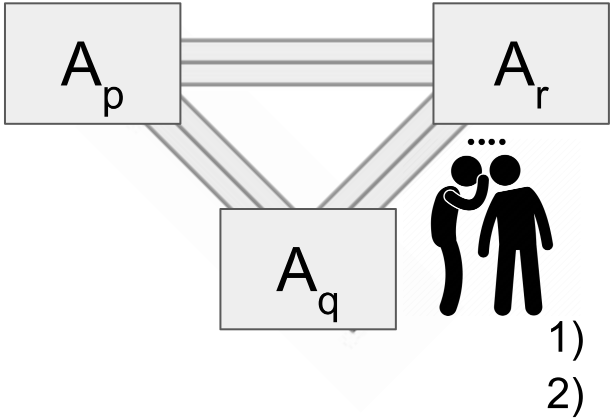
A_r



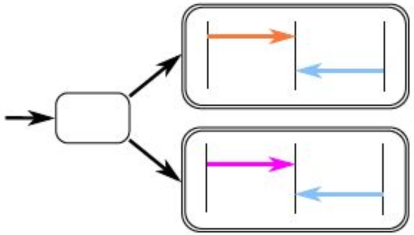
MST monolithic protocol refinement

PSPACE-hard, non-compositional

Monolithic protocol refinement = *given* a CSM, *sub*protocol fidelity + deadlock freedom



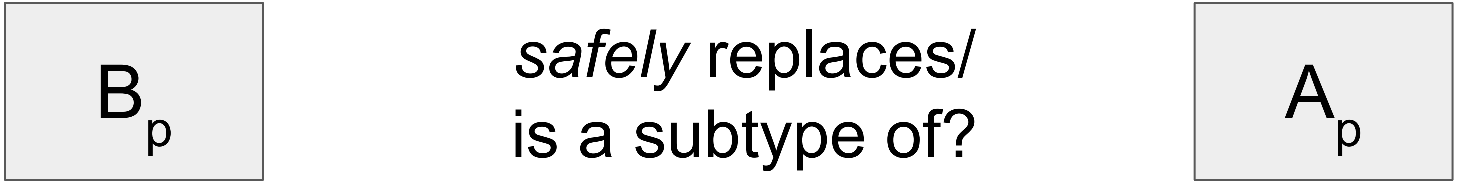
protocol refines?



1) CSM language ⊆ global type language
2) CSM is deadlock-free

MST subtyping

Subtyping = can B_p *safely* replace A_p?



B_p

safely replaces/
is a subtype of?

A_p

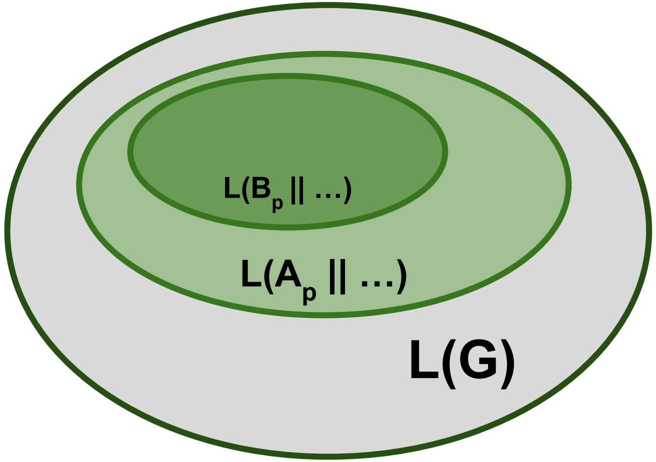
MST subtyping: existing work

Subtyping = can B_p *safely* replace A_p?

Our work provides:

- A stronger notion of safety:

  language inclusion + deadlock freedom



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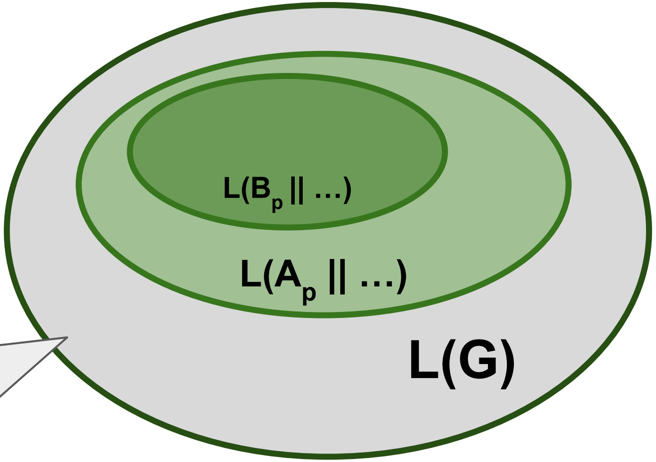
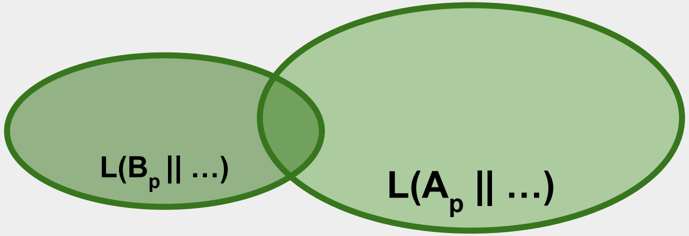


Brief digression on terminology:

"Asynchronous subtyping"



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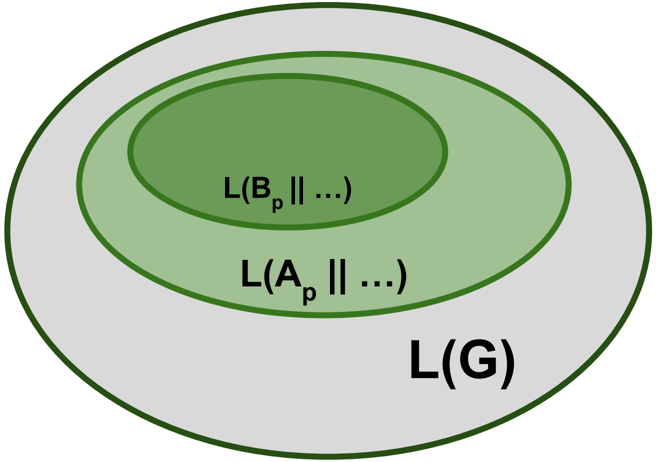


Subtyping = can B_p *safely* replace A_p?

Our work provides:

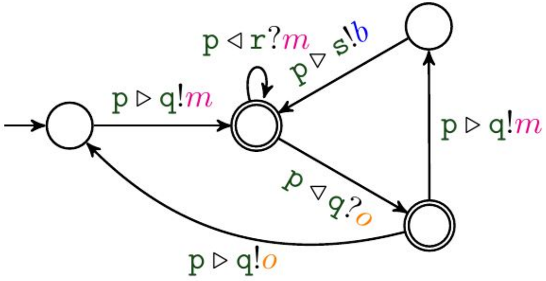
- A stronger notion of safety:

  language inclusion + deadlock freedom

- A more expressive implementation model

  than [Lange and Yoshida, TACAS'16] [Ghilezan et al., POPL'21]

  - Mixed choice
  - Final states with outgoing transitions
  - Unrestricted control flow



MST subtyping: existing work

Subtyping = can B_p *safely* replace A_p?

Our work provides:

- A stronger notion of safety:

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- A more expressive implementation model

  than [Lange and Yoshida, TACAS'16] [Ghilezan et al., POPL'21]

  - Mixed choice
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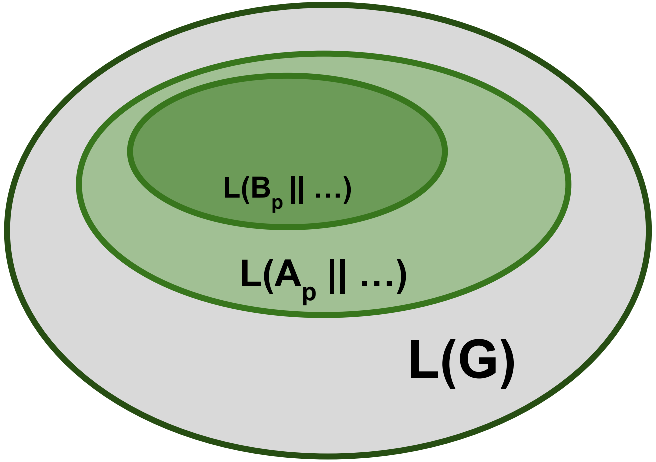
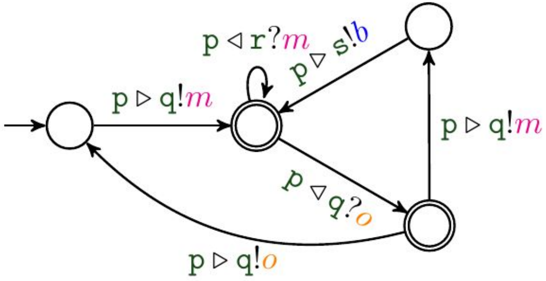
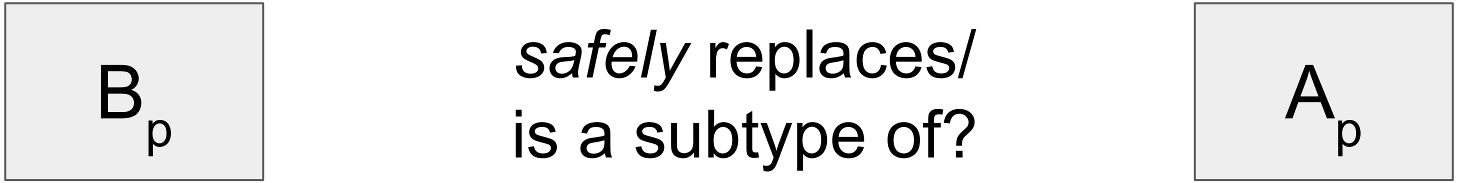
- A **context-dependent** subtyping relation





MST subtyping

Subtyping = can B_p *safely* replace A_p?



B_p

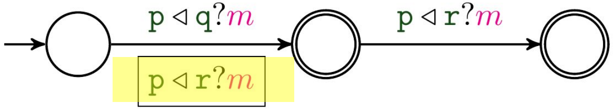
safely replaces/
is a subtype of?

A_p

Folkloric subtyping: *"add receives, remove sends"*

MST subtyping

Subtyping = can B_p *safely* replace A_p?



safely replaces/
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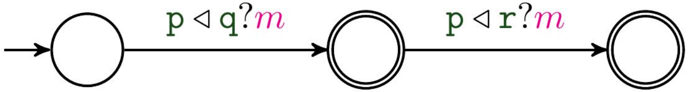
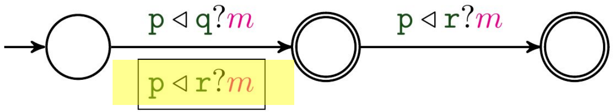
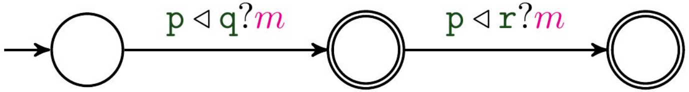


MST subtyping

Subtyping = can B_p *safely* replace A_p?



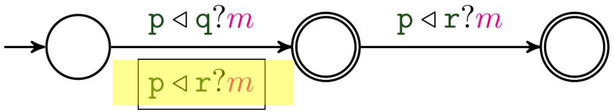
safely replaces/
is a subtype of?



It depends!

MST subtyping

Subtyping = can B_p *safely* replace A_p?



safely replaces/
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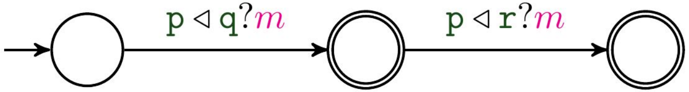
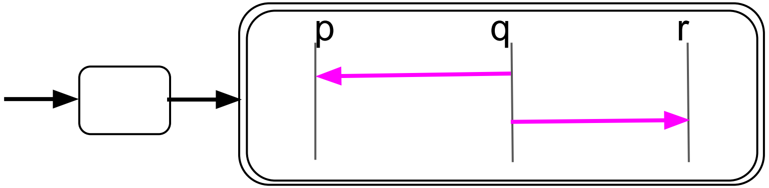
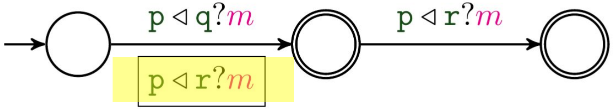


It depends! ✓

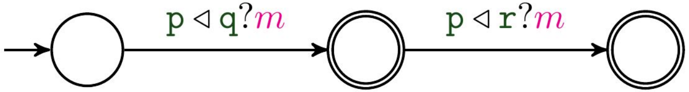


MST subtyping

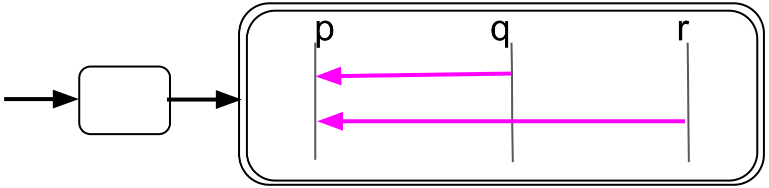
Subtyping = can B_p *safely* replace A_p?



safely replaces/
is a subtype of?



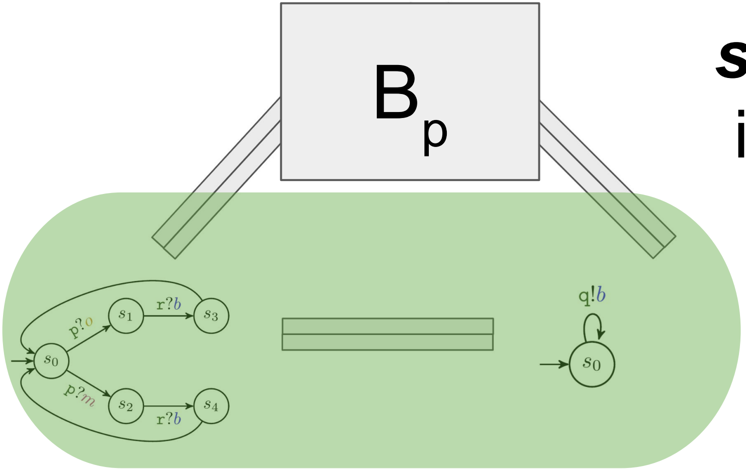
It depends! ✗



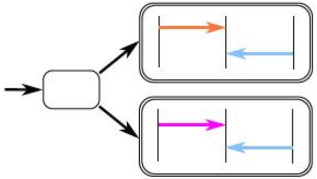
MST protocol refinement (subtyping)

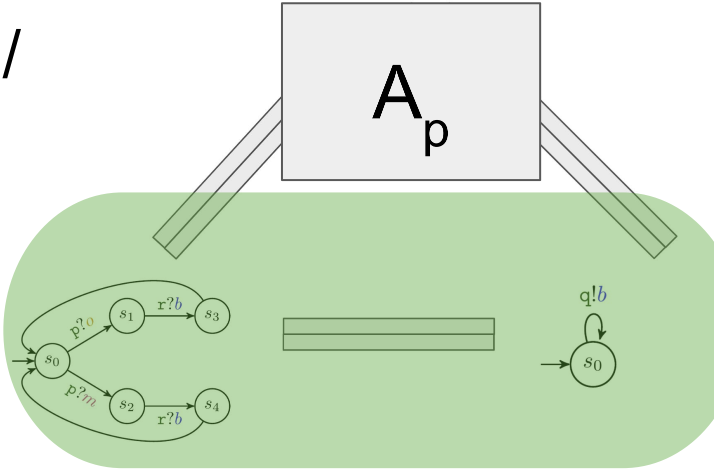
polynomial, compositional, context-dependent

Protocol refinement = **for all well-behaved contexts under G**, can B_p safely replace A_p?



safely$_G$ replaces/
is a subtype of?





A_p
s_0
s_1
s_2
s_3
s_4
p?o
p?m
r?b
r?b
q!b
s_0